List of Errata for the Book

A Mathematical Introduction to Compressive Sensing

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This list was last updated on May 8, 2015. If you see further errors, please send us an e-mail at foucart@math.uga.edu and rauhut@mathc.rwth-aachen.de.

Chapter 2

- Page 51, Theorem 2.15: the statement concerns $s$-sparse vectors, not $2s$-sparse vectors.
- Page 51, Line 22: ‘$\hat{p} \ast \hat{x} = \hat{p} \cdot x = 0$’ should read ‘$\hat{p} \ast \hat{x} = N \hat{p} \cdot x = 0$’.
- Page 52, Line 11: ‘so that the trigonometric polynomial $q$ vanishes on $S$’; this statement is only valid if the support of $x$ is exactly $S$; to repair the argument, take $\hat{q}(1), \ldots, \hat{q}(s)$ as a solution of the linear system with a maximum number of consecutive zero values for $\hat{q}(s), \hat{q}(s-1), \ldots$ (this is done by solving a sequence of linear systems), then replace $s$ by $\|x\|_0$ and $S$ by $\text{supp}(x)$ in Lines 9-12.

Chapter 3

- Page 74, Exercise 3.4: the condition about the invertibility of the submatrices is not necessary.
- Page 74, Exercise 3.8: one may assume that the matrix $A \in \mathbb{C}^{m \times N}$ is of full rank $m < N$.
- Page 75, Exercise 3.10: a complex conjugation is missing on line 8, which should read
  \[ \Delta_n = \|A(x^{n+1} - x^n)\|_2^2 = \bar{x}_{jn+1}^{n+1} (A^*(y - A x^n))_{jn+1} \]

Chapter 4

- Page 109, Exercise 4.20(b): one should read ‘$M \in \mathbb{C}^{n_1 \times n_2}$’ instead of ‘$M \in \ker A \setminus \{0\}$’; the occurrences ‘$\|e\|_2$’ and ‘$\|A(Z) - y\|_2$’ of an $\ell_2$-norm should be replaced by ‘$\|e\|$’ and ‘$\|A(Z) - y\|$’ with a general norm; and ‘quadratically constrained’ should be rephrased as ‘inequality-constrained’.
Chapter 5

• Page 120, Theorem 5.12: ‘For $m \geq 3$’ should read ‘For $m > 3$’ (indeed, when $m = 3$, equiangular systems of $N = m(m+1)/2$ vectors in $\mathbb{R}^m$ exist — see Exercise 5.5 — yet $m + 2$ is not the square of an odd integer); in the proof of the theorem, one should also verify that $\Sigma_1$ and $\Sigma_2$ are nonzero, but if they were, then $\Sigma_1 - (\sqrt{m + 1})\Sigma_2 = 0$ would mean that $\sqrt{m + 2} = 1/c$ is the other eigenvalue of $B$, namely $(N/m - 1)/c = ((m+1)/2 - 1)/c$, which is impossible when $m > 3$.

Chapter 6

• Page 134, Line 7: ‘the interval $[1 - \delta_s, 1 + \delta_s]$’ should read ‘the interval $[\sqrt{1 - \delta_s}, \sqrt{1 + \delta_s}]$’

• Page 134, Line 14: ‘relative $\ell_2(\mathbb{R})$’ should read ‘relative to $\ell_2(\mathbb{R})$’

• Pages 139-140, Proof of Theorem 6.8: more care is required to deal with the fact that the last block $A_n$ may have less than $t$ columns — one should establish $\text{tr}(H) \geq N(1 - \delta_s)$ instead of (6.10) and $\text{tr}(H)^2 \leq mN ((n - 1)\delta_s^2 + (1 + \delta_s)^2)$ instead of (6.11), while the rest of the argument remains unchanged.

• Page 142, Line 15: replace ‘Corollary 4.5’ by ‘Theorem 4.5’

• Page 161, Lines 15 and 16: $\delta_{s+n}$ should be $\delta_{s+s^0+n}$ — this implies that $\delta_{s+K}$ found in Lines 19, 21, 24, as well as on Page 163, Lines 4 and 5, should be $\delta_{s+s^0+K}$, but there is no repercussion on the final result because $\alpha/\gamma < 1$ still holds.

• Page 171, Exercise 6.7: replace ‘the unit ball in $\ell_p$’ by ‘the unit ball in $\ell_p^N$’

• Page 173, Exercise 6.19: establish the result under the condition $\delta_{3s} < 1/2$, not $\delta_{3s} < 1/3$

• Page 173, Exercise 6.21: assume that all vectors and matrices are real-valued rather than complex-valued throughout the exercise.

Chapter 7

• Page 190, Line 11: the extra parenthesis after $(-B_{\ell})$ should be removed, so that it reads $\exp(\theta X_{\ell}) = f(X_{\ell}) = f(t(-B_{\ell}) + (1-t)B_{\ell}) \leq \ldots$

• Page 191, Line 15: it should read ‘from Hoeffding’s inequality (Theorem 7.20)’

• Page 199, Line 1: ‘Bernstein’s inequality’ instead of ‘Bernstein inequality’
• Page 199, Exercise 7.3: the exponent 2 on the right-hand side of the desired inequality has to be replaced by \( \frac{p}{p-1} \), so that it reads
\[
\mathbb{P} \left( \left| \sum_{\ell=1}^{M} a_{\ell} X_{\ell} \right| > t \| a \|_2 \right) \geq c_p \frac{(\sigma^2 - t^2)^{\frac{p}{p-1}}}{\mu^{2\frac{p}{p-1}}}, \quad 0 \leq t \leq \sigma.
\]

• Page 199, Exercise 7.6: the inequality to be proved is in fact
\[
\mathbb{E} \exp \left( \frac{tX^2}{2c} \right) \leq \frac{1}{\sqrt{1 - 2t}}
\]
which is valid for any (not necessarily nonnegative) \( t \leq 1/2 \)

Chapter 8

• Page 219, Line 10: replace ‘positive semidefinite’ by ‘positive definite’, so that it reads ‘... is concave on the set of positive definite matrices.’

Chapter 9

• Page 289, Line 1: an expectation \( \mathbb{E} \) is missing; the left-hand side of the inequality should read
\[
\mathbb{E} \min_{z \in \mathcal{N}(x)} \| g - z \|_2^2
\]

• Page 306, Fig. 9.2: the caption should include ‘Image courtesy of Jared Tanner’ instead of ‘Image Courtesy by Jared Tanner’

• Page 306, Exercise 9.2: the inequality inside the probability should be strict, otherwise (9.61) is wrong for \( x = 0 \)

• Page 307, Exercise 9.6: a renormalization is missing — it is indeed the matrix \( \sqrt{\frac{\pi/2}{m}} A \) that satisfies the stated modified restricted isometry property.

Chapter 10

• Page 312, Line 8: there is a deplorable break at the end of this line — \( \lim_{m \to \infty} d^m(K, X) = 0 \) should appear as one block

• Page 313, Line 25: ‘\( = 0 \)’ is missing after \( \lambda_{2,0}(v) \), so that one should read \( \lambda_{2,\lambda_1(v)} = \lambda_{2,0}(v) = 0 \)

• Page 314, Line 23: ‘quasi-triangle’ should be ‘quasi-triangle inequality’
Chapter 13

- Page 439, Lemma 13.4: replace lines 2 and 3 by
  For each \( i \in R(S) \), let \( \ell(i) \in S \) denote a fixed left vertex connected to \( i \). Then the set
  \[
  E'(S) := \{ j \in E(S) : j \neq \ell(i) \} = E(S) \setminus \{ \ell(i)i, i \in R(S) \}
  \]

- Page 442, Line 3: ‘\( j = \text{card}(R(J)) \)’ should read ‘\( j = \text{card}(J) \)’

- Page 453, Line 1: replace \( m \) by \( m' \) in ‘given \( y \in \mathbb{C}^m \), ...’

- Page 453, Line 5: it should be emphasized that the stated condition may not be met if the bipartite graph fails to be a lossless expander, so the algorithm is not well defined in this case

- Page 454, Lines 2, 3, 4, 6: \( B_{k,j}, B_{k,j}^*, B_{\ell+1,j}^* \) should instead be \( B_{k,j}', B_{k,j}'^*, B_{\ell+1,j}'^* \)

- Page 454, Line 7: the two sums should start at \( k = 1 \) and not at \( k = 0 \)

Appendix B

- Page 544, Theorem B.4: ‘interiors’ should read ‘relative interiors’

- Page 545, Remark B.5: instead of \( K_1, K_2 \) intersecting in only one point, the second application of Theorem B.4 requires that \( K_1, K_2 \) intersect in only one point not in the relative interior of \( K_1 \), i.e., \( K_1 \cap K_2 = \{ x_0 \} \) with \( x_0 \not\in \text{ri}(K_1) \)

References

- Page 614, Reference 503: ‘Wakinm’ should be ‘Wakin’

Back cover

- Line 12: ‘build’ should be ‘built’