Analysis of Sparse Recovery in MIMO Radar

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***Abstract*—We study a multiple-input multiple-output (MIMO) model for radar and provide recovery guarantees for a compressive sensing approach. Several transmit antennas send random pulses over some time-period and the echo is recorded by several receive antennas. The radar scene is resolved on an azimuth-range-Doppler grid. Sparsity is a natural assumption in this context and we study recovery of the radar scene via ℓ_1-minimization. On the one hand we provide an estimate for the well-known restricted isometry property (RIP) ensuring stable and robust recovery. Compared to standard estimates available for Gaussian random measurements we require more measurements in order to resolve a scene of certain sparsity. Nevertheless, we show that our RIP estimate is optimal up to possibly logarithmic factors. By turning to a nonuniform analysis for a fixed radar scene, we reveal that the fine-structure of the support set (not only its size) influences the recovery performance. By introducing a parameter measuring the well-behavedness of the support we derive a bound for the number of measurements sufficient for recovery that resembles the minimal one for Gaussian random measurements if this parameter is close to optimal, i.e., if the support set is not pathological. Our analysis complements earlier work due to Friedlander and Strohmer where the support set was assumed to be random.**

I. Introduction

MIMO (multiple-input multiple-output) radar uses multiple antennas to simultaneously transmit several different waveforms and multiple antennas which record the reflected signals. Modeling the radar scene on a grid in azimuth-range-Doppler space may lead to a high-dimensional vector of reflectivities that needs to be recovered from the measured data and often one ends up with an underdetermined linear system that has to be solved for that purpose. In many cases, it is natural to assume that only a few targets are present leading to sparsity of the reflection coefficients. This motivates to consider compressive sensing techniques [1] such as ℓ_1-minimization in order to accurately recover the target scene.

The aim of our paper is to provide a theoretical analysis of sparse recovery via ℓ_1-minimization in MIMO radar. We use the same measurement model as Friedlander and Strohmer in [2], where a first analysis has been conducted assuming randomly distributed support sets. On the one hand, we provide an estimate for the restricted isometry property of the measurement matrix. Compared to standard estimates available for Gaussian random matrices for instance, we require more measurements for a given sparsity. Nevertheless, we show that our bound is optimal. On the other hand, we also provide a nonuniform recovery result where we introduce a parameter depending on the fine structure of the support set. For support sets with minimal parameter (i.e., support sets which are “well-balanced”, see below), the required number of measurements resembles the expected one from standard compressive sensing up to additional logarithmic factors. To the best of our knowledge, it has not been observed earlier for realistic measurement matrices in compressive sensing that the recovery performance may depend on the fine structure of the support set.

Notation. Below, $\|\mathbf{x}\|_p = (\sum_j |x_j|^p)^{1/p}$ denotes the usual ℓ_p-norm of a vector $\mathbf{x}$, for $1 \leq p < \infty$. Moreover, $\operatorname{supp}(\mathbf{x}) = \{\ell : \mathbf{x}_\ell \neq 0\}$ denotes the support of $\mathbf{x}$.

II. MIMO Measurement Model

Let us describe the MIMO radar measurement model from [2], see also [3]–[5] for a similar model (with random antenna locations, however). We have N_T transmit antennas and N_R receive antennas that are co-located on a line with equidistant spacings, i.e., the transmit antennas occupy the locations $(0, j d_T \lambda) \in \mathbb{R}^2$, $j = 0, \ldots, N_R - 1$, while the receive antennas are located in $(0, k d_R \lambda)$, $k = 0, \ldots, N_T - 1$, where d_R and d_T denote the corresponding spacings and λ denotes the wavelength of the carrier frequency of the radar system. The transmit antennas send signals $s_j(t)$, $j = 1, \ldots, N_T$, that are modeled as independent periodic, continuous-time white Gaussian noise with period-duration T and bandwidth B. Sampling at the Nyquist-rate results in finite-length vectors $\mathbf{s}_j(\ell) = s_j(\ell \Delta_t)$, $t = 1, \ldots, N_t$ with $\Delta_t = \frac{1}{2B}$ and $N_t = T/\Delta_t$. Then the $\mathbf{s}_j$ are independent standard complex Gaussian random vectors in $\mathbb{C}^{N_t}$.

Suppose that a target is present at distance r, azimuth angle θ (i.e., at position $(r\cos(\theta), r\sin(\theta)) \in \mathbb{R}^2$) and radial speed ν having unit reflectivity. We assume that the target is far away from the radar device, i.e., $r \gg \max\{\lambda d_T N_T, \lambda d_R N_R\}$. Then it (approximately) produces the following return at receiver k,

$$y_k(t) = \sum_{j=1}^{N_T} s_j(t - 2r/c) \times e^{2\pi i c \lambda^{-1}[t - c^{-1}(2\nu \cdot t + 2r + \sin(\theta) j d_T \lambda + \sin(\theta) k d_R \lambda)]}, \tag{1}$$

see also eq. (2) in [4]. Here, c is the speed of light. In the following we assume that the targets are located on a grid in azimuth-range-Doppler space, i.e., $(\frac{2r}{c}, \sin(\theta), \lambda^{-1}\nu) \in G$ with

$$G = \{(\tau_0 + \Delta_\tau \tau, \beta_0 + \Delta_\beta \beta, f_0 + \Delta_f f) : (\tau, \beta, f) \in \mathcal{G}\},$$
$$\mathcal{G} := [N_\tau] \times [N_\beta] \times [N_f],$$

where $[N] := \{1, 2, \ldots, N\}$. Let us further introduce the translation and modulation operators on $\mathbb{C}^{N_t}$, for $\mathbf{z} \in \mathbb{C}^{N_t}$, as

$$(T_\tau \mathbf{x})_\ell = \mathbf{x}_{\ell-\tau}, \qquad (M_f \mathbf{x})_\ell = e^{2\pi i f \ell / N_t} \mathbf{x}_\ell,$$

where subtraction is understood modulo N_t. Note that we may assume $N_\tau \leq N_t$ and $N_f \leq N_\tau$ because otherwise we face ambiguities as $T_{\tau+N_t} = T_\tau$ and $M_{f+N_\tau} = M_f$. For simplicity, we will actually set

$$N_\tau = N_f = N_t$$

in the remainder of the paper, but note that our results will also hold for smaller values of N_f and N_τ.

For $\beta \in [N_\beta]$, the so-called array manifolds are given as

$$\mathbf{a}_T(\beta) = \begin{pmatrix} 1 \\ e^{2\pi i d_T \Delta_\beta \beta} \\ \vdots \\ e^{2\pi i d_T \Delta_\beta \beta (N_T - 1)} \end{pmatrix} \in \mathbb{C}^{N_T}$$

and

$$\mathbf{a}_R(\beta) = \begin{pmatrix} 1 \\ e^{2\pi i d_R \Delta_\beta \beta} \\ \vdots \\ e^{2\pi i d_R \Delta_\beta \beta (N_R - 1)} \end{pmatrix} \in \mathbb{C}^{N_R}.$$

Moreover, for $(\tau, f) \in [N_t] \times [N_t]$ we consider the matrix $\mathbf{S}_{\tau,f} \in \mathbb{C}^{N_t \times N_T}$ matrix with columns $M_f T_\tau \mathbf{s}_k \in \mathbb{C}^{N_t}$, $k = 1, \ldots, N_T$.

After demodulation (multiplication by $e^{-2\pi i c \lambda^{-1} t}$) of the signals y_j in (1) and sampling at the Nyquist rate $\Delta_t = \frac{1}{2B}$, the measured data at receiver j corresponding to a target at grid position (τ, β, f) – related to the physical location $\big(\frac{c}{2}(\tau_0 + \Delta_\tau \tau), \arcsin(\beta_0 + \Delta_\beta \beta), \lambda(f_0 + \Delta_f f)\big)$ – take the form $\mathbf{y}_k = (\mathbf{a}_R(\beta))_k \mathbf{a}_T(\beta)^T \mathbf{S}_{\tau,f}^T \in \mathbb{C}^{N_t}$. The signal collecting all time-sampled data for all receive antennas corresponding to the grid position $(\tau, \beta, f) \in [N_t] \times [N_\beta] \times [N_t]$ is then given as

$$\mathbf{Z}(\tau, \beta, f) = \mathbf{a}_R(\beta) \mathbf{a}_T(\beta)^T \mathbf{S}_{\tau,f}^T \in \mathbb{C}^{N_R \times N_t}.$$

Now, let $\mathbf{x} \in \mathbb{C}^{N_t \times N_\beta \times N_t}$ be a vector representing the reflectivities at the grid points. If no target is present then the corresponding entry of $\mathbf{x}$ is zero, so that when only few targets are present the vector $\mathbf{x}$ is sparse. The measurements (indexed by the receive antennas and time) of the full target scene are then given by

$$\mathbf{y} = \sum_{\tau=1}^{N_t} \sum_{\beta=1}^{N_\beta} \sum_{f=1}^{N_t} \mathbf{x}_{\tau,\beta,f} \mathbf{Z}(\tau, \beta, f) \in \mathbb{C}^{N_R \times N_t}.$$

In practice, noise is present so that the measurements can be written in matrix-vector notation as

$$\operatorname{vec}(\mathbf{y}) = \mathbf{A}\mathbf{x} + \mathbf{n} \in \mathbb{C}^{N_R \cdot N_t} \tag{2}$$

where the columns of the measurement matrix $\mathbf{A} \in \mathbb{C}^{N_R \cdot N_t \times N_t^2 \cdot N_\beta}$ are the vectors $\operatorname{vec}(\mathbf{Z}(\tau, \beta, f)) \in \mathbb{C}^{N_R \cdot N_t}$, $(\tau, \beta, f) \in [N_t] \times [N_\beta] \times [N_t]$, and $\mathbf{n} \in \mathbb{C}^{N_R \cdot N_t}$ represents a noise vector. Note that the number $m = N_R \cdot N_t$ of measurements is significantly smaller than the dimension $N = N_t^2 \cdot N_\beta$ of the vector $\mathbf{x}$ representing the target scene. Reconstructing $\mathbf{x} \in \mathbb{C}^N$ from the measurements $\mathbf{y} \in \mathbb{C}^m$ becomes then the task of solving a highly underdetermined system of linear equations. Taking into account that sparsity of $\mathbf{x}$ is a natural assumption in many radar applications, we follow a compressed sensing approach in order to reconstruct $\mathbf{x}$. Note that since the transmit signals $\mathbf{s}_j$, $j = 1, \ldots, N_T$, are Gaussian random vectors, the resulting matrix $\mathbf{A}$ is a highly structured random matrix. While a first analysis of this matrix has been conducted in [2] for random support sets, we aim at deepening the understanding of its properties in the context of compressed sensing.

As in [2], we will assume below that the parameters take the values

$$\Delta_\tau = \Delta_t = \frac{1}{2B}, \quad \Delta_f = \frac{1}{T}$$
$$\Delta_\beta = \frac{2}{N_R N_T}, \quad N_\beta = N_R N_T,$$

and $d_T = \frac{1}{2}, d_R = N_T/2$ or $d_T = N_R/2, d_R = 1/2$. In particular, the choice of Δ_β (together with the results below) means that the angular resolution is the same as the one of an array with $N_R N_T$ antennas. This is a huge gain compared to the resolution of $\frac{2}{N_R + N_T}$ which one would usually obtain for a radar device with $N_T + N_R$ antennas.

III. Compressed sensing

In general, compressed sensing aims at reconstructing vectors $\mathbf{x} \in \mathbb{C}^m$ from underdetermined linear measurements $\mathbf{y} = \mathbf{A}\mathbf{x}$ with $\mathbf{A} \in \mathbb{C}^{m \times N}$, where $m \ll N$, see e.g. [1] for an introduction. Assuming that $\mathbf{x}$ is sparse, i.e., $\|\mathbf{x}\|_0 = \#\{\ell : \mathbf{x}_\ell \neq 0\} \leq s$ (or $\mathbf{x}$ is at least approximately sparse), we can reconstruct $\mathbf{x}$ via several tractable algorithms including ℓ_1-minimization. The latter consists in solving the convex optimization problem

$$\min_{\mathbf{z}} \|\mathbf{z}\|_1 \quad \text{subject to } \mathbf{A}\mathbf{z} = \mathbf{y}.$$

Several approaches for analyzing ℓ_1-minimization have been developed. The restricted isometry property of the measurement matrix $\mathbf{A}$ is a well-known tool for this task. For a sparsity $s \leq N$, the restricted isometry constant δ_s of $\mathbf{A}$ is defined as the smallest constant such that

$$(1 - \delta_s)\|\mathbf{x}\|_2^2 \leq \|\mathbf{A}\mathbf{x}\|_2^2 \leq (1 + \delta_s)\|\mathbf{x}\|_2^2 \quad \forall \mathbf{x} \text{ s.t. } \|\mathbf{x}\|_0 \leq s.$$

If $\delta_{2s} < 1/\sqrt{2}$ then ℓ_1-minimization reconstructs every s-sparse $\mathbf{x}$ from $\mathbf{y} = \mathbf{A}\mathbf{x}$, see [1], [6]. Moreover, reconstruction is stable under sparsity defect and perturbation of the measurements by noise when employing the noise-constraint ℓ_1-minimization problem

$$\min_{\mathbf{z}} \|\mathbf{z}\|_1 \quad \text{subject to } \|\mathbf{A}\mathbf{z} - \mathbf{y}\|_2 \leq \varrho.$$

In fact, if $\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{n}$ with $\|\mathbf{n}\|_2 \le \varrho$ and $\delta_{2s} < 1/\sqrt{2}$ then the minimizer $\mathbf{x}^\sharp$ of the above optimization problem satisfies

$$\|\mathbf{x} - \mathbf{x}^\sharp\|_2 \le C_1 \varrho + \frac{C_2}{\sqrt{s}} \inf_{\|\mathbf{z}\|_0 \le s} \|\mathbf{x} - \mathbf{z}\|_1.$$

We will also consider the LASSO problem below which consists in solving the regularized problem

$$\min_{\mathbf{z}} \frac{1}{2}\|\mathbf{A}\mathbf{z} - \mathbf{y}\|_2^2 + \lambda\|\mathbf{z}\|_1, \tag{3}$$

where $\lambda > 0$ is a suitable regularization parameter. We will present a nonuniform recovery result for LASSO which holds for a fixed sparse vector with high probability. For the corresponding analysis, we will rely on conditions ensuring the success of support recovery via the LASSO derived in [7].

While it is very hard (and by-now open) to analyze the restricted isometry property for deterministic measurement matrices in the optimal parameter regime, it is known that Gaussian random matrices satisfy $\delta_s \le \delta$ with high probability provided that

$$m \ge C\delta^{-2} s \ln(eN/s). \tag{4}$$

Similar guarantees (both for the RIP as well as for nonuniform recovery) with some additional logarithmic factors have been derived also for practically more relevant structured random matrices such as random partial Fourier matrices [8]–[10], partial random circulant matrices [10]–[12], time-frequency structured random matrices [12], [13] and a structured matrix arising from another radar measurement setup [14].

IV. Random supports

Friedlander and Strohmer [2] analyzed sparse recovery (and support recovery) for the MIMO measurement matrix in (2) in connection with LASSO when the support set of the sparse vector follows a uniform distribution among all support sets in $[N_t] \times [N_\beta] \times [N_t]$. Additionally, they assume that the signs of the nonzero entries follow a Steinhaus distribution (each entry is uniformly distributed on the complex unit torus), but the magnitudes are arbitrary. Under a few (not so important) assumptions on the parameters N_t and N_β, they consider the solution $\mathbf{x}^\sharp$ of the LASSO problem (3) with $\lambda = 2\sigma\sqrt{2\log(N_t^2 N_\beta)}$ computed from $\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{n}$ where $\mathbf{n}$ is a complex Gaussian random vector whose with i.i.d. mean-zero and variance σ^2. If the absolute entries of $\mathbf{x}$ are above a certain threshold determined by σ, then they show that the supports of $\mathbf{x}^\sharp$ and $\mathbf{x}$ coincide provided that

$$N_R \cdot N_t \ge Cs\log(N_t^2 N_\beta). \tag{5}$$

(Note that Theorem 5 in [2] contains a typo in this estimate.) Below we will study sparse recovery without assuming randomness of the support.

V. RIP Estimate

Our first main result establishes the RIP for the radar measurement matrix in (2).

Theorem 1. *Let the transmit signals $\mathbf{s}_j \in \mathbb{C}^{N_t}$, $j = 1, \ldots, N$, be independent standard complex Gaussian random vectors. For $\delta, \varepsilon \in (0,1)$, assume that*

$$N_t \ge C\delta^{-2} s \max\left\{\log^2(N)\log^2(s), \log(1/\varepsilon)\right\} \tag{6}$$

where $N = N_t^2 N_\beta$. Then the scaled MIMO radar measurement matrix $\frac{1}{\sqrt{N_T N_R N_t}}\mathbf{A} \in \mathbb{C}^{N_R N_t \times N}$ satisfies $\delta_s \le \delta$ with probability at least $1 - \varepsilon$.

Clearly, the above theorem implies stable s-sparse recovery via ℓ_1-minimization with high probability provided that

$$N_t \gtrsim s\log^2(N)\log^2(s). \tag{7}$$

The proof writes the restricted isometry constant δ_s as the supremum of a chaos process and then uses tools developed in [12] which ultimately require to bound certain covering numbers.

Compared to the standard estimate (4), our new bound (7) requires more measurements even when ignoring the logarithmic factors since here the number of measurements is $m = N_R N_t$, so that the factor N_R is missing on the left hand side of (7). Nevertheless, our bound (7) is optimal (up to possibly logarithmic factors) due to the special structure of the measurement matrix. This can be seen by considering a certain (scaled) submatrix $\mathbf{B} \in \mathbb{C}^{N_t \times N_t^2}$ of $\mathbf{A}$ for which one can argue that the corresponding restricted isometry constant satisfies $\delta_s(\mathbf{B}) \le \delta_s(\mathbf{A})$, see [15] for details. A general lower bound for restricted isometry constants [1, Corollary 10.8] shows then that necessarily

$$N_t \ge C_\delta s\log(eN_t^2/s)$$

if $\delta_s(\mathbf{A}) \le \delta$ and hence $\delta_s(\mathbf{B}) \le \delta$. This means that (7) is optimal up to logarithmic factors.

VI. Nonuniform recovery

We now pass to a nonuniform analysis of the recovery performance of the MIMO measurement matrix $\mathbf{A}$ introduced in (2). This means that we fix a sparse vector $\mathbf{x}$, and show that recovery is successful with high probability on the draw of $\mathbf{A}$. It turns out that the fine structure of the support set of $\mathbf{x}$ plays a significant role.

In order to describe this phenomenon we introduce an equivalence relation on the set of angle parameters $\beta \in [N_\beta]$, where we recall that $N_\beta = N_R N_T$. We say that $\beta, \beta' \in [N_\beta]$ are equivalent, $\beta \sim \beta'$, if

$$\beta - \beta' \in N_R\mathbb{Z}.$$

For a given support set $S \subset [N_t] \times [N_\beta] \times [N_t]$ we define

$$S_{[\beta]} = \{(\tau', \beta', f') \in S : \beta' \sim \beta\}.$$

Definition 1. *A support set S is called η-balanced if for all equivalence classes $[\beta]$ of angle parameters it holds*

$$|S_{[\beta]}| \le \eta\frac{|S|}{N_R}.$$

Since there are N_R angle classes, the parameter η ranges between 1 and N_R. Support sets which are η-balanced with a small (constant) parameter η are favourable for sparse recovery as shown by the next result.

Theorem 2. *Let* $\mathbf{x}$ *be an* s*-sparse target scene with* η*-balanced support. Let the MIMO measurement matrix* $\mathbf{A} \in \mathbb{C}^{N_R N_t \times N}$*,* $N = N_t^2 N_\beta$*, be generated by the transmit signals* $\mathbf{s}_j$*,* $j = 1, \ldots, N_T$*, chosen to be independent complex standard Gaussian random vectors. Further, assume that the signs of the nonzero entries of* $\mathbf{x}$ *form a Steinhaus sequence (i.e., the entries are independent and uniformly distributed on the complex torus). Take measurements*

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{n},$$

where the entries of the noise vector $\mathbf{n}$ *are independent normal distributed with mean-zero and variance* σ^2*. Let* $\mathbf{x}^\sharp$ *be the solution of the LASSO problem* (3) *with* $\lambda = \frac{2\sigma}{\sqrt{N_T N_R N_t}} \sqrt{2\log(N)}$*. If*

$$N_R N_t \geq C\eta s \log^3(\eta N/\varepsilon) \tag{8}$$

and

$$\min_{\Theta \in \mathrm{supp}(\mathbf{x})} |\mathbf{x}_\Theta| > \frac{8\sigma}{\sqrt{N_T N_R N_t}} \sqrt{2\log(N)},$$

then, with probability at least $1 - \varepsilon$*,*

$$\mathrm{supp}(\mathbf{x}^\sharp) = \mathrm{supp}(\mathbf{x}).$$

For η-balanced supports with $\eta \leq c$, we hence recover the usual scaling of the number of measurements in terms of the sparsity (up to additional logarithmic factors) common in compressed sensing. In the worst case that $\eta = N_R$, we obtain essentially the same scaling as for the RIP estimate in (6). The theorem also explains the discrepancy between Friedlander and Strohmer's earlier result (5) for random supports and the estimate (6) for the RIP. In fact, a randomly chosen support set will be η-balanced for small η with high probability.

The proof of the above theorem uses results in [7] on support recovery via LASSO. A crucial ingredient consists in proving that the measurement matrix with columns restricted to $S_{[\beta]}$ is well-conditioned under a suitable condition on N_t. To this end, one relies on tools such as symmetrization, decoupling and a version of the noncommutative Khintchine inequality for chaos, see [10, Theorem 6.22]. We again refer to [15] for details.

Preliminary numerical experiments confirm the dependence of the fine structure of the support sets on the recovery performance as indicated by (8). Detailed experiments will be described in [15].

Rather than support recovery we note that one may also derive reconstruction error estimates in the spirit of the main result in [14].

VII. Conclusion

We have studied sparse recovery for a MIMO measurement setup. We provided both an estimate for the restricted isometry property and a nonuniform recovery result. The latter reveals a dependence of the recovery performance on the fine structure of the support set and explains the discrepancy between the RIP estimate and a previous estimate for random support sets due to Friedlander and Strohmer [2]. To the best of our knowledge such a dependence on fine properties of the support has not been observed earlier in the compressed sensing literature.

Our approach assumes that the targets are located on the grid points of the discretization. This may not be completely realistic in practice. It remains as an open question to study the effect of off-grid targets on the reconstruction error and to possibly develop methods which may overcome the shortcomings of working with a discrete location grid.

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