Time-Varying Linear Systems: Identification and Transmission through Unidentified Channels

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Outline

1. Introduction: Linear Time-Varying Systems
2. Identification under Side Constraints
3. Signal Transmission over Unidentified Channels
4. Example: Message Transmission with unknown Channel Support
Time-Varying Linear Systems
Time-varying Linear SISO Systems

   Linear time-varying SISO channels are described by operators of $H : L^2(\mathbb{R}) \rightarrow L^2(\mathbb{R})$ the form

$$
(Hf)(t) = \int_{\mathbb{R} \times \mathbb{R}} \eta_H(\tau, \nu) \cdot f(t - \tau) e^{i2\pi \nu t} d\nu d\tau = \int_{\mathbb{R} \times \mathbb{R}} \eta_H(\tau, \nu) (M_\nu T_\tau f)(t) d\nu d\tau
$$

with

- **Spreading function**: $\eta_H : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{C}$
- **Translation (time-shift) operator**: $(T_\tau f)(t) = f(t - \tau)$
- **Modulation (frequency shift)**: $M_\nu (f)(t) = f(t) e^{i2\pi \nu t}$
Multiple-Input Multiple-Output Systems (MIMO):
Channels with $N$-inputs and $M$-outputs are characterized by operators $H : (L^2(\mathbb{R}))^N \rightarrow (L^2(\mathbb{R}))^M$ of the form

$$H \begin{pmatrix} f_1 \\ \vdots \\ f_N \end{pmatrix} = \begin{pmatrix} H_{1,1} & \cdots & H_{1,N} \\ \vdots & \ddots & \vdots \\ H_{M,1} & \cdots & H_{M,N} \end{pmatrix} \begin{pmatrix} f_1 \\ \vdots \\ f_N \end{pmatrix} = \begin{pmatrix} \sum_{n=1}^N H_{1,n} f_n \\ \vdots \\ \sum_{n=1}^N H_{M,n} f_n \end{pmatrix}.$$ 

Each subchannel $H_{m,n}$ is a TVL SISO system.
Finite-dimensional TVL Channels

The identification problem of TVL systems $H : (L^2(\mathbb{R}))^N \to (L^2(\mathbb{R}))^M$ can be reduced to a finite-dimensional problem $H : (\mathbb{C}^L)^N \to (\mathbb{C}^L)^M$

$$H \begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix} = \begin{pmatrix} H_{1,1} & \cdots & H_{1,N} \\ \vdots & \ddots & \vdots \\ H_{M,1} & \cdots & H_{M,N} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix} = \begin{pmatrix} \sum_{n=1}^N H_{1,n}x_n \\ \vdots \\ \sum_{n=1}^N H_{M,n}x_n \end{pmatrix}.$$ 

wherein each sub-system $H_{n,m} : \mathbb{C}^L \to \mathbb{C}^L$ has the form

$$H_{m,n}x = \sum_{\ell=0}^{L-1} \sum_{k=0}^{L-1} \eta_{m,n}(k, \ell) M^\ell T^k x = G(x)\eta$$

with spreading coefficients $\{\eta_{m,n}(k, \ell)\}_{k,\ell=0}^{L-1}$, and with the translation operator $T : \mathbb{C}^L \to \mathbb{C}^L$ and the modulation operator $M : \mathbb{C}^L \to \mathbb{C}^L$ given by

$$T : \begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_{L-1} \end{pmatrix} \mapsto \begin{pmatrix} x_{L-1} \\ x_0 \\ \vdots \\ x_{L-2} \end{pmatrix} \quad \text{and} \quad M : \begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_{L-1} \end{pmatrix} \mapsto \begin{pmatrix} x_0 e^{i2\pi \cdot \frac{1}{L}} \\ x_1 e^{i2\pi \cdot \frac{1}{L}} \\ \vdots \\ x_{L-1} e^{i2\pi \cdot \frac{1}{L} \cdot (L-1)} \end{pmatrix}.$$
Operator Paley–Wiener spaces

Set of all linear operators $\mathbb{C}^L \to \mathbb{C}^L$ is spanned by time-frequency shifts $M^\ell T^k$

$$\mathcal{L}(\mathbb{C}^L) = \left\{ H = \sum_{k=0}^{L-1} \sum_{\ell=0}^{L-1} \eta(k, \ell) M^\ell T^k : \eta(k, \ell) \in \mathbb{C} \text{ for all } (k, \ell) \in \mathbb{Z}_L \times \mathbb{Z}_L \right\}$$

SISO Operator Paley–Wiener space: For $\Lambda \subset \mathbb{Z}_L \times \mathbb{Z}_L$

$$\text{OPW}(\Lambda) = \text{span} \left\{ M^\ell T^k : (k, \ell) \in \Lambda \right\}$$

MIMO Operator Paley–Wiener space: For $\Lambda = \{\Lambda_{m,n}\}_{m,n=1}^{M,N}$ with $\Lambda_{m,n} \in \mathbb{Z}_L \times \mathbb{Z}_L$

$$\text{OPW}(\Lambda) = \{ H : H_{m,n} \in \text{OPW}(\Lambda_{m,n}) \}$$
Identification under linear side constraints
Identification – SISO

Definition (Identifiable)
The space \( OPW(\Lambda) \) is identifiable if and only if there exists an identifier \( c \in \mathbb{C}^L \) such that for each \( H \in OPW(\Lambda) \) the equation

\[
y = Hc = \sum_{(k,\ell) \in \Lambda} \eta(k,\ell) M^\ell T^k c = G(c) \eta
\]

is uniquely solvable for \( \eta \in \mathbb{C}^\Lambda \).

Remark
\( G(c) \) is Gabor matrix of size \( L \times L^2 \)

\[
G(c) = \begin{bmatrix} M^0 T^0 c , M^0 T^1 c , \cdots , M^{L-1} T^{L-1} c \end{bmatrix}
\]

Theorem (Identification of SISO Channels)
The space \( OPW(\Lambda) \) is identifiable if and only if \( |\Lambda| \leq L \).
Identification – MIMO

Definition (Identifiable MIMO)
The space $OPW(\Lambda)$ is identifiable if and only if there exist vectors $c = (c_1, c_2, \ldots, c_N) \in (\mathbb{C}^L)^N$ such that for each $H \in OPW(\Lambda)$ the map $H \mapsto y = Hc$ is injective.

Theorem (Identification of MIMO Channels)
The space $OPW(\Lambda)$ is identifiable if and only if

$$\sum_{n=1}^{N} |\Lambda_{m,n}| \leq L \quad \text{for every } m = 1, \ldots, M.$$

Assumptions

- Subchannels $H_{m,n}$ and their time-frequency components $\eta_{m,n}(k, \ell)$ are independent
- Information about one channel does not help to identify another.
Linear Constrains Between Spreading Coefficients

▷ Channels $\mathbf{H}$ in $OPW(\Lambda) \subset \mathcal{L}(\mathbb{C}^L)$:

$$
y = \mathbf{H}\mathbf{c} = \sum_{(k,\ell) \in \Lambda} \eta(k, \ell) \mathbf{M}^{\ell} \mathbf{T}^k \mathbf{c} = \mathbf{G}(\mathbf{c}) \eta
$$

▷ $OPW(\Lambda)$ is identifiable if $|\Lambda| \leq L$.

▷ Assume linear relations between the spreading coefficients are known

$$
\sum_{k,\ell} \alpha_{k,\ell} \eta(k, \ell) = \beta \quad \text{for some} \quad \alpha_{k,\ell}, \beta \in \mathbb{C}
$$

Intuition/Question

- Let $A\eta = b$ be a given set of $M \geq 1$ linear independent side constraints.
- Let $OPW_{A,b}(\Lambda)$ be the set of all $\mathbf{H} \in OPW(\Lambda)$ which satisfy these side constraints.
- $OPW_{A,b}(\Lambda)$ is identifiable if and only if $|\Lambda| \leq L + M$?
Linear Relations between Time-Frequency Components

▷ In the SISO case, linear relations between the spreading coefficients of the channel are expressed by

\[ b = A \eta \]

▷ Including the equation for channel identification yields

\[
\begin{bmatrix}
  y \\
  b
\end{bmatrix} =
\begin{bmatrix}
  G(c) \\
  A
\end{bmatrix} \eta
\]

\[ \rightarrow \text{More equations for the same number of unknowns should help to identify the channel.} \]

Theorem (One linear side constraint)

Let \( H \in OPW(\Lambda) \) with \( \Lambda \subset \mathbb{Z}_L \times \mathbb{Z}_L \) with \( |\Lambda| = L + 1 \) and let \( a \in \mathbb{C}^{L+1}, a \neq 0 \). There there exists a \( c \in \mathbb{C}^L \) so that

\[
\begin{bmatrix}
  G(c) |_{\Lambda} \\
  a^*
\end{bmatrix}
\]

is invertible. Thus the channel coefficients \( \eta \) are identifiable. Moreover, the set of all such identifiers \( c \) constitute a dense open subset of \( \mathbb{C}^L \).
More Side Constrains

Lemma (No general solution for more than one side constraint)

Let $\mathbf{H} \in \text{OPW}(\Lambda)$ with $\Lambda \subset \mathbb{Z}_L \times \mathbb{Z}_L$ with $|\Lambda| > L + 1$. There exist matrices $\mathbf{A}$ of size $(|\Lambda| - L) \times |\Lambda|$ with $\mathbf{A} \neq 0$ such that there is no $\mathbf{c} \in \mathbb{C}^L$ such that the matrix

$$
\begin{bmatrix}
\mathbf{G}(\mathbf{c})|_{\Lambda} \\
\mathbf{A}
\end{bmatrix}
$$

has full rank.

Theorem (Sufficient condition for identifiably)

Let $\mathbf{H} \in \text{OPW}(\Lambda)$ with $\Lambda \in \mathbb{Z}_L \times \mathbb{Z}_L$ of size $|\Lambda| = R > L$. Assume that there exists a subset $\tilde{\Lambda} \subset \Lambda$ of size $L$ so that

(i) $\tau_j(\Lambda) = \tau_j(\tilde{\Lambda})$ whenever $\tau_j(\Lambda) \neq 0$.

(ii) $\text{ind}(\tau') \neq \text{ind}(\tau(\Lambda))$ for every $L$-tubel $\tau' \preceq \tau(\tilde{\Lambda})$ of size $L$ different from $\tau(\Lambda)$.

Given any full spark matrix $\mathbf{A}$ of size $(R - L) \times R$, then there exists an identifier $\mathbf{c} \in \mathbb{C}^L$ so that the $R \times R$ matrix

$$
\begin{bmatrix}
\mathbf{G}(\mathbf{c})|_{\Lambda} \\
\mathbf{A}
\end{bmatrix}
$$

is invertible. Moreover, the set of all such identifiers $\mathbf{c}$ constitute a dense open subset of $\mathbb{C}^L$. 

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Signal Transmission over Unidentified Channels
Motivation

- Two step procedure for data transmission over frequency-selective channels
  1. Estimate (identify) the channel $H \in \mathcal{H} \subset L(\mathbb{C}^L)$. 
  2. Transmit data $x$ from a certain data set $\mathcal{X} \subset \mathbb{C}^L$. Data recovery at the receiver using estimated channel.

- In time-varying channels, this procedure has to repeated regularly, to update channel state information.
- In rapidly changing channels, two step procedure becomes more and more inefficient.

Transmission through unidentified channel

- Combine channel identification and signal recovery.
- Transmission scheme $y = H(x + c)$ with
  - $H \in \mathcal{H} \subset L(\mathbb{C}^L)$ is a **unknown channel** from a known subset $\mathcal{H}$.
  - $x \in \mathcal{X}$ is the **data signal** from a certain data set $\mathcal{X} \subset \mathbb{C}^L$.
  - $c \in \mathbb{C}^L$ is a **pilot signal** (designed based on the knowledge of $\mathcal{H}$ and $\mathcal{X}$).

Problem
Find (necessary and/or sufficient) conditions on $\mathcal{H}$ and $\mathcal{X}$ such that there exists a pilot $c \in \mathbb{C}^L$ such that every $x \in \mathcal{X}$ can uniquely be recovered form $y = H(x + c)$ for any unknown channel $H \in \mathcal{H}$.
Relation to Blind Deconvolution

\[ \mathcal{H} = \text{OPW}(\Lambda) \text{ with } \Lambda \{(0, 0), (1, 0), \ldots, (L, 0)\} \subset \mathbb{Z}_L \times \mathbb{Z}_L \]

\[ y = Hc = \sum_{k=0}^{L-1} \eta(k, 0) T^k x \]

- Recover \( x \in \mathcal{X} \subset \mathbb{C}^L \) and \( \eta \) from \( y \) (without knowing \( \eta \)).
- \( x \) and \( \eta \) assumed to be sparse.
Conditions for Data Recovery and/or Channel Identification

Natural Conditions

(I) Identifiability of $H$: The map $H \mapsto Hc$ is injective on $H$.

(R) Recovery condition for known channel: Every $H \in H$ is injective on $X$.

Subsets of $\mathbb{C}^L$

$Hc = \{Hc : H \in H\} : \text{All possible output vectors for the pilot } c.$

$HX = \{Hx : H \in H, x \in X\} : \text{Possible output of arbitrary data vector in } X.$

Further Conditions

(i) $\text{span}\{Hc\} \cap \text{span}\{HX\} = \{0\} : \text{Isolate } Hc \text{ and } Hx \text{ from the channel output } y = H(c + x).$

(ii) $H(X + c) \cap H'(X + c)$ for every $H \neq H'$ in $H : \text{Identify } H \text{ from } y = H(c + x) \text{ with unknown } x \in X.$

(iii) $H(x + c) = H'(x' + c)$ implies $x = x' : \text{Guarantees exact recovery of } x \in X \text{ but not identification } H$

$(i) \xrightarrow{(I)} (ii) \xrightarrow{(R)} (iii)$
Degrees of Freedom

- $\mathcal{H} = OPW(\Lambda) \subset \mathcal{L}(\mathbb{C}^L)$ is a linear subspace of dimension $|\Lambda|$.
- Assume $\mathcal{X} \subset \mathbb{C}^L$ is a linear subspace of dimension $K$.
- Counting degrees of freedom, we must have

$$|\Lambda| + K \leq L \quad \text{and} \quad |\Lambda| \leq L. \quad (1)$$

as a necessary condition for exact recovery of $H \in \mathcal{H}$ and $x \in \mathcal{X}$.
- Without identifying $H$, we may get $K > L - |\Lambda| \implies \text{how?}$
- When we get equality in (1)?
Example: 1-Dimensional Signal Space

▷ For a given (and known) $\Lambda \subset \mathbb{Z}_L \times \mathbb{Z}_L$, we consider the operator Paley–Wiener space

$$\mathcal{H} = \text{OPW}(\Lambda) = \left\{ H = \sum_{k, \ell \in \Lambda} \eta(k, \ell) M^\ell T^k : \eta(k, \ell) \in \mathbb{C} \right\} \subset \mathcal{L}(\mathbb{C}^L)$$

▷ Assume a 1–dimensional signal space $\mathcal{X} = \text{span}\{v\} \subset \mathbb{C}^L$ for some $v \in \mathbb{C}^L$.

▷ For $x = uv \in \mathcal{X}$, with $u \in \mathbb{C}$, the received signal is

$$y = H(x + c) = uH(v) + H(c) = G(c)|_{\Lambda} \eta + uG(v)|_{\Lambda} \eta$$

▷ Separation of $Hx$ and $Hc$ from $y \implies$ span$\{Hc\} \cap$ span$\{Hv\} = \{0\} \implies$ rang$[G(c)|_{\Lambda}] \perp$ rang$[G(v)|_{\Lambda}]$.

For rang$[G(v)|_{\Lambda}] = \text{span}\{a\}$ is a 1-dimensional subspace, we have the following result.

**Theorem**

Let $\Lambda \subset \mathbb{Z}_L \times \mathbb{Z}_L$ with $1 \leq |\Lambda| \leq L - 1$ and let $a \in \mathbb{C}^L \setminus \{0\}$ be arbitrary. Then there exists a vector $c \in \mathbb{C}^L$ such that the $L \times |\Lambda| + 1$ matrix $[G(c)|_{\Lambda}, a]$ has full rank.
General Construction

- The set of all time-frequency shifts \( \{ M^\ell T^k \} _{\ell,k=0}^{L-1} \) can be separated into \( L + 1 \) commutative subgroups \( G_s = \{ M^{2rk} T^k : k = 0, 1, \ldots, L - 1 \} \), \( s = 0, 1, \ldots, L - 1 \)
  \( G_L = \{ M^k : k = 0, 1, \ldots, L - 1 \} \)

- Each commutative subgroup \( G_s \) posses a set of common eigenvectors (i.e. the chirp sequences) \( e_s(\ell) : \ell = 0, 1, \ldots, L - 1 \)
  which forms on orthogonal basis for \( \mathbb{C}^L \).

Theorem

Let \( \Lambda \subset \mathbb{Z}_L \times \mathbb{Z}_L \) such that there exists an \( s \in \{0, 1, \ldots, L\} \) so that \( \Lambda \subset G_s \).
There exists a subspace \( \mathcal{X}^c = \text{span}\{e_s(1), \ldots, e_s(K)\} \subset \mathbb{C}^L \) of dimension \( K = L - |\Lambda| \) and a \( c \in \mathbb{C}^L \) such that

\[
\text{span}\{ \mathcal{H} c \} \cap \text{span}\{ \mathcal{H} \mathcal{X}^c \} = \{0\}
\]

and such that Conditions (I) and (R) are satisfied.
Maximum Size of Data Subspace

Given a subspace $\mathcal{X}$ of dimension $K$, it is desirable that the dimension of $\text{span}\{H\mathcal{X}\}$ is again $K$.

For a one-dimensional $\mathcal{X} = \text{span}\{v\}$, we have $\dim(\text{span}\{H\mathcal{X}\}) = \text{rank}(G(v)|_\Lambda)$.

Question: Given a support set $\Lambda \subset \mathbb{Z}_L \times \mathbb{Z}_L$ of $\mathcal{H} = \text{OPW} (\Lambda)$ with $|\Lambda| \leq L$. What is the minimum rank of $G(v)|_\Lambda$ for $v$ varying in $\mathbb{C}^L$?

Theorem
Let $L \geq 3$ be an odd integer and $\Lambda \subset \mathbb{Z}_L \times \mathbb{Z}_L$ with $|\Lambda| \leq L$. Then

$$\min_{v \in \mathbb{C}^L \setminus \{0\}} \text{rank}(G(v)|_\Lambda) \leq N(\Lambda)$$

with

$$N(\Lambda) = 1 + \min_{s \in \{0,1,\ldots,L\}} \min \left\{|I| : I \subset \mathbb{Z}_L \times \mathbb{Z}_L \text{ with } \Lambda \subset I + \mathcal{G}_s\right\},$$

and where $I + \mathcal{G}_s = \{x + y : x \in I, y \in \mathcal{G}_s\}$. 

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Example with Unknown Channel Support
Channel Model

▷ Time continuous Time-Varying-Linear SISO Channel $H : L^2(\mathbb{R}) \rightarrow L^2(\mathbb{R})$:

$$g(t) = (Hf)(t) = \int \int_{\mathbb{R} \times \mathbb{R}} \eta_H(\tau, \nu) \cdot f(t-\tau) e^{i2\pi \nu t} d\nu d\tau = \int \int_{\mathbb{R} \times \mathbb{R}} \eta_H(\tau, \nu) (M\nu T f)(t) d\nu d\tau$$

▷ Time discrete Time-Varying-Linear SISO Channel $H : L^2(\mathbb{R}) \rightarrow L^2(\mathbb{R})$:

$$g(t) = (Hf)(t) = \sum_{k=0}^{K-1} \sum_{\ell=0}^{M-1} \eta_H(k, \ell) \cdot f(t-k\Delta \tau) e^{i2\pi \ell \Delta \nu t} = \sum_{k=0}^{K-1} \sum_{\ell=0}^{M-1} \eta_H(k, \ell) \cdot f(t-kT) e^{i2\pi \ell T L t}$$

▷ Rectification of the channel support region:

with $\Delta \tau = T$ and $\Delta \nu = \frac{1}{TL}$ for some prime $L \geq 5$. 
Transmitted Signal

▷ Transmitted signal: Delta train followed by a guard interval

\[ f(t) = \sum_{m=0}^{2L-1} x_m \delta(t - mT) \quad \text{with} \quad x_m = \begin{cases} 
\text{data symbol} & : 0 \leq m \leq L - 1 \\
0 & : L \leq m \leq 2L - 1
\end{cases} \]

▷ Received signal:

1. Sampling at rate \(1/T\):

\[ g_n = (Hf)(nT), \; n = 0, 1, \ldots, 2L - 1. \]

2. Add two consecutive blocks of length \(L\):

\[ y_n = g_n + g_{n+L}, \; n = 0, 1, \ldots, L - 1. \]

▷ Write in vector form:

\[ y = Hx = \sum_{\ell=0}^{L-1} \sum_{k=0}^{L-1} \eta(k, l) M^\ell T^k x \]

with \(x = (x_0, \ldots, x_{L-1})^T\) and \(y = (x_0, \ldots, x_{L-1})^T\) and with \(H \in OPW(\Lambda)\).

Remark: Using a periodic weighted delta train

\[ f(t) = \sum_{p \in \mathbb{Z}} \sum_{m=0}^{2L-1} x_m \delta(t - mT - pLT) \]

results in a similar expression as in (2) but requires periodic data.
Transmission Scheme

▷ **Aim:** We want to transmit a message \( \gamma = \{ \gamma_1, \ldots, \gamma_Q \} \subset \mathbb{C} \) of size \( Q \leq L \) over the channel \( H \in OPW(\Lambda) \) and recover \( \gamma \) at the receiver without knowing \( H \) and without knowing the channel support \( \Lambda \) in advance.

▷ **Data symbols:** The data symbols a sum of a pilot and the actual message

\[
x = c + \sum_{q=1}^{Q} \gamma_q e_q
\]

with the pilot signal \( c \) which is chosen to be an *Alltop sequence* and \( e_q \) are particular *chirp sequences*

\[
c(n) = \frac{1}{\sqrt{L}} \exp \left( i \frac{2\pi}{L} n^3 \right) \quad \text{and} \quad e_{mL+r}(n) = \frac{1}{\sqrt{L}} \exp \left( i \frac{2\pi}{L} [r + mn + rn^2] \right), \quad n \in \mathbb{Z}_L
\]

▷ **Received signal:**

\[
y = Hx = \sum_{\ell=0}^{L-1} \sum_{k=0}^{L-1} \eta(k, \ell) M^\ell T^k x = G(x) \eta = G(c) \eta + U \eta = \Phi \begin{pmatrix} \eta \\ s \end{pmatrix}
\]

with a \( L \times 2L^2 \) measurement matrix \( \Phi = [G(c), U] \) and a sparse vector \( s = f(\eta, \gamma) \).

▷ **Recovery of the message:** Solve CS problem (3), then determine \( \gamma = g(\eta, s) \).
Compressive Sampling Problem

Compressive Sampling Problem of size $L \times 2L^2$

$$y = \Phi \left( \begin{array}{c} \eta \\ s \end{array} \right) \quad \text{with} \quad |\text{supp}(\eta, s)^T| \leq (1 + Q)|\Lambda|$$

Lemma

The coherence of the measurement matrix $\Phi = [G(c), U]$ is upper bounded by $\mu(\Phi) \leq \frac{2}{\sqrt{L}}$.

Remark: Welsh bound is $\frac{1}{\sqrt{L+1}}$.

Theorem

Let $H \in OPW(\Lambda) \subset \mathcal{L}(\mathbb{C}^L)$ be an unknown channel with unknown support set $\Lambda$ where $L \geq 5$ is a prime number. For any message $\gamma \in \mathbb{C}^Q$ can be transmitted over $H$ and recovered at the receiver.

$$Q \leq \frac{\sqrt{L}}{4|\Lambda|} - 1$$
Numerical Experiment

- \( L = 307 \)
- OMP
- average over 100 channels (random support, Gaussian coefficients)
- red line: 1% rel. error
- simulation much better than bound
Summary
Current and Future Work

- **Identification of SISO and MIMO TVL Channels:**
  - Identification of stochastic channels and stochastic sequences
  - Conditions on support of the covariance of the spreading function $\eta$.
  - Linear side constraints in terms of covariance.

- **Transmission over Unidentified Channels:**
  - Stochastic encoding, RIP
  - Scheme which maximum transmission rate
  - Continuous time setting
Related Publications


